



Optimize HPC Applications on Heterogeneous Architectures

Deliverable: D5.1.7.2 (L2.7.2)

Tests of hardware and software developments

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- Authors: CEA/DAM / Guillaume Colin de VERDIERE,

Dassault Aviation / Quang Vinh DINH

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HISTORY

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1 Summary

This deliverable is the implementation of the tests specification described in deliverable D5.1.7.1. It covers basic functional tests of the Cirrus platform, tests of its accelerator component, a more elaborate benchmark by the Dassault Aviation (DA) partner and a specific focus on the tools developed by the UVSQ partner.

2 Introduction

This deliverable describes the results of the tests specified in deliverable D5.1.7.1, which were accomplished through a collaborative work between the partners CEA, DA and UVSQ. In this deliverable we will first show that the prototype of an accelerator-based architecture, namely the Cirrus platform, is functional, second that it is performant on different use cases and third that tools developed by the UVSQ partner can help to optimize codes on such an architecture.

In addition, we will describe a more elaborate investigation allowed by the prototype. This benchmark involves a full FEM mini-application using industrial-grade simulation data, provided by DA. Even though it was not funded during the November 2013 – December 2014 time period of the H4H/Perfcloud project, DA was willing to participate in order to test its mini-application on the Xeon Phi architecture of the Cirrus platform.

3 Specific tests

3.1 Code selected

To test the machines as well as the environments, we used the Hydro benchmark [Github].

This basic benchmark is available in a number of implementations allowing for rapid evaluation of a platform, be it a single node or a cluster with or without acceleration.

3.2 Basic test of the machine

The first test will be to use the OpenCL + MPI version of Hydro. It should produce the same output as on a production cluster. The output should be identical on the first 10 iterations. This run will validate the OS installation as well as the network linking the nodes (if any).

The first step was to install a functional operating system which was done without problem (see Table 1).

```
[coling@cirrus50 Src]$ uname -a
Linux cirrus50 2.6.32-279.14.1.el6_lustre.x86_64 #1 SMP Fri Dec 14 23:22:17 PST 2012 x86_64 x86_64 x86_64
GNU/Linux
[coling@cirrus50 Src]$ cat /etc/redhat-release
CentOS release 6.3 (Final)
[coling@cirrus50 Src]$
```



Table 1: Operating system of the Cirrus prototype

Then we had to install numerous packages to make sure to have a versatile prototype.

| [coling@cirrus50 Src]\$ mo | dule avail | | | |
|---|---|-------------------|--------------|-------------------|
| | /opt/Modules/defau | ult/modulefiles/a | applications | |
| abinit/7.2.2 | cp2k/2.2 | gromacs/4.6.3 | 3(default) | openfoam/2.2.2 |
| abinit/7.2.2_gpu | cp2k/2.3_xc | gromacs/4.6.3 | 3_gpu | openfoam/2.3.0 |
| abinit/7.4.2 | cp2k/2.4 | gromacs/4.6.5 | 5 | salome/7.2.0 |
| abinit/7.4.2_gpu | desmond/3.4.0.2 | gromacs/4.6.5 | 5_gpu | saturne/2.0.1 |
| abinit/7.6.1 | espresso/4.2.1 | gromacs/4.6.5 | 5_plumed | saturne/3.0.0 |
| abinit/7.6.1_gpu | espresso/5.0.1_cuda | gromos/1.2.0 | | turbomole/6.4 |
| abinit/7.6.2_gpu | espresso/5.0.2 | lammps/23Feb1 | 13_phonon | wgrib/1.8.1.0b |
| abinit/7.6.2_mpi(default) | freefem++/3.16 | lammps/7jun13 | 3 | wrf/3.3 |
| aster/11.3.0 | freefem++/3.23 | nco/3.9.4 | | wrf/3.3.1 |
| bigdft/1.6.0 | gamess/2013.05 | nco/4.0.5 | | wrf/3.3_cuda |
| <.> | | | | |
| gnuplot/4.6.0 lib | tiff/3.9.4 plot | icus/2.42 | wxx11/2.8. | 12 |
| gnuplot/4.6.5 lib | tiff/4.0.3 pyqt/ | /4.9.1 | xmgrace/5. | 1.23 |
| | /opt/Modules/de: | fault/modulefiles | s/parallel - | |
| bullxmpi/1.1.16.5(default |) mpi/ir | ntelmpi/5.0.1.035 | 5 | |
| cuda/4.2(default) | mpi/mp | pc/2.4.0 | | |
| cuda/5.0 | mpi/mp | pc/2.5.0 | | |
| hmpp/3.3.0 | mpi/or | penmpi/1.6.5 | | |
| intelmpi/4.0.3.008 mpi/openmpi/1.8.2 | | | | |
| intelmpi/4.1.0.030(defaul | intelmpi/4.1.0.030(default) mpiprofile/bullxmpi/big_collective_io | | | |
| | | | | |
| lapack/intel/13.1.3.192 tbb/14.0.2.144(default) | | | | |
| lapack/intel/14.0.0.080 | tbb/15.0.0.0 | 90 | | |
| | /opt/Modules/defa | ault/modulefiles, | /compilers - | |
| c++/14.0.2.144(default) | c/intel/12.1.7 | .256 | fortran/ | pgi/12.6 |
| c++/gnu/3.4.6 | c/intel/12.1.9 | .293 | fortran/ | pgi/13.4 |
| c++/gnu/4.4.5 | c/intel/13.1.3 | .192 | fortran/ | pgi/14.3 |
| c++/gnu/4.5.1 | c/intel/14.0.0 | .080 | gnu/3.4. | 6 |
| c++/gnu/4.6.3 | c/intel/14.0.1 | .106 | gnu/4.4. | 5 |
| c++/gnu/4.8.1 | c/intel/14.0.2 | .144(default) | gnu/4.5. | 1 |
| c++/gnu/4.9.0 | c/intel/15.0.0 | .090 | gnu/4.6. | 3 |
| c++/intel/11.1.072 | c/pgi/11.10 | | gnu/4.8. | 1 |
| c++/intel/12.1.7.256 | c/pgi/12.6 | | gnu/4.9. | 0 |
| c++/intel/12.1.9.293 | c/pgi/13.4 | | intel/11 | .1.072 |
| c++/intel/13.1.3.192 | c/pgi/14.3 | | intel/12 | .1.7.256 |
| c++/intel/14.0.0.080 | fortran/14.0.2 | .144(default) | intel/12 | .1.9.293 |
| c++/intel/14.0.1.106 | fortran/gnu/3.4 | 1.6 | intel/13 | .1.3.192 |
| c++/intel/14.0.2.144(defa | ult) fortran/gnu/4.4 | 4.5 | intel/14 | .0.0.080 |
| c++/intel/15.0.0.090 | fortran/gnu/4.5 | 5.1 | intel/14 | .0.1.106 |
| c++/pgi/11.10 | fortran/gnu/4.6 | 6.3 | intel/14 | .0.2.144(default) |



| c++/pgi/12.6 | fortran/gnu/4.8.1 | intel/15.0.0.090 |
|-------------------------|--|-----------------------|
| c++/pgi/13.4 | fortran/gnu/4.9.0 | java/openjdk/1.6.0_24 |
| c++/pgi/14.3 | fortran/intel/11.1.072 | java/oracle/1.7.0_25 |
| c/14.0.2.144(default) | fortran/intel/12.1.7.256 | jdk/1.7.0_25 |
| c/gnu/3.4.6 | fortran/intel/12.1.9.293 | llvm/3.1 |
| c/gnu/4.4.5 | fortran/intel/13.1.3.192 | llvm/3.2 |
| c/gnu/4.5.1 | fortran/intel/14.0.0.080 | pgi/11.10 |
| c/gnu/4.6.3 | fortran/intel/14.0.1.106 | pgi/12.6 |
| c/gnu/4.8.1 | <pre>fortran/intel/14.0.2.144(default)</pre> | pgi/13.4 |
| c/gnu/4.9.0 | fortran/intel/15.0.0.090 | pgi/14.3 |
| c/intel/11.1.072 | fortran/pgi/11.10 | |
| [coling@cirrus50 Src]\$ | | |

Table 2: (truncated) listing of all packages made available on the prototype

At this stage, the prototype is usable by different types of testers. Here, we will focus on the KNC accelerator. The tests were made using the OpenCL version of the Hydro benchmark. The nice feature of OpenCL is that, when properly programmed, software using this language can run on different architectures without modifications.

The following listing (Table 3) illustrates what to expect for a run using two MPI tasks of the OpenCL version of Hydro on a classical (and modern) piece of hardware. This output will be our reference run.



Hydro: OpenCL compute unit type = CPU HydroC: Simple decomposition HydroC: nx=2 ny=1 +-----|nx=2000 |nv=2000 |nxystep=128 |tend=1600.000 |nstepmax=10 |noutput=0 |dtoutput=0.000 Hydro starts in double precision. [0/ 2] x= 0 X=1000 y= 0 Y=2000 / u= -1 d= -1 l= -1 r= 1 [1/ 2] x=1000 X=2000 y= 0 Y=2000 / u= -1 d= -1 l= 0 r= -1 Nb platform : 1 [0] Profile : FULL PROFILE
[0] VERSION : OpenCL 1.2 LINUX [0] NAME : Intel(R) OpenCL
[0] VENDOR : Intel(R) Corporation [0] EXTENSIONS : cl_khr_fp64 cl_khr_icd cl_khr_global_int32_base_atomics cl_khr_global_int32_extended_atomics Hydro: 001 has 0 GPU Hydro: 001 has 0 ACC Hydro: 001 has 1 CPU Hydro: 001 has 1 CPU (0) :: device maxcu 56 mxwkitdim 3 mxwkitsz 1024 1024 1024 mxwkgsz 1024 mxclockMhz 2300 mxmemallocsz 32237 (Mo) globmemsz 128951 (Mo) type 2 [CPU] extensions: cl_khr_fp64 cl_khr_icd cl_khr_global_int32_base_atomics cl_khr_global_int32_extended_atomics cl_khr_local_int32_base_atomics cl_khr_local_int32_extended_atomics cl_khr_byte_addressable_store cl_intel_printf cl_ext_device_fission cl_intel_exec_by_local_thread Device 0 supports double precision floating point prefered vector size: c=1 s=1 i=1 l=1 f=1 d=1 Hydro: 000 has 0 GPU Hydro: 000 has 0 GPU Hydro: 000 has 0 ACC Hvdro: 000 has 1 CPU [0] : nbdevices = 1 Building a CPU version Build OpenCL (opts="-cl-mad-enable -DNVIDIA -DHASFP64 -I/ccc/ghome/ocre/coling/Github/Hydro/HydroC/oclHydroC_2D/Src ") OK. Centered test case : [1] 2 1002 Hydro 0: initialize acc 0.083604s Hydro 1: initialize acc 0.108037s Hydro starts main loop. Hydro starts main loop. --> step=1 1.33631e-03, 1.33631e-03 (0.263s) * --> step=2 2.67261e-03, 1.33631e-03 (0.374s) --> step=3 5.70914e-03, 3.03653e-03 (0.300s) * --> step=4 8.74568e-03, 3.03653e-03 (0.313s) --> step=5 1.24942e-02, 3.74854e-03 (0.351s) * --> step=6 1.62428e-02, 3.74854e-03 (0.351s) (11.4 MC/s) --> step=7 2.06309e-02, 4.38811e-03 (0.327s) (11.2 MC/s) * --> step=8 2.50190e-02, 4.5811e-03 (0.3316s) (12.7 MC/s) * --> step=9 2.95700e-02, 4.55100e-03 (0.316s) (12.7 MC/s) * Hydro ends in 00:00:05.700s(5.700) without init: 3.249s. (DP] Hydro ends in 00:00:05.700s(5.700) without init: 3.249s. [DP] GATCON CONPRI EOS SLOPE TRACE QLEFTR QLEFTR RIEMAN CMPFLX UPDCON COMPDT MAKBOU ALLRED MIN 0.230371 0.204546 0.170753 0.174949 0.248487 0.243243 0.346009 0.235095 0.375060 0.086611 0.183798 0.000094 MAX 0.339994 0.270067 0.213999 0.268457 0.318193 0.400195 0.485847 0.331852 0.403947 0.087503 0.600992 0.316446 0.237306 0.192376 0.221703 0.283340 0.321719 0.415928 0.283474 0.389504 0.087057 AVG 0.285182 0.392395 0.158270 Average MC/s: 12.3

Table 3: a typical output of the Hydro code on a reference machine (here a node based on a dual socket Haswell-EP with 14 cores at 2.3 GHz each) using two MPI tasks



3.3 Test of the accelerator

To validate the proper functioning of an accelerator, we will use the OpenCL version of Hydro. The virtue of OpenCL is that it is present on any kind of accelerator nowadays. We compare the output of Hydro with a reference run on a cluster (the one of Table 3). Both the OpenCL versions should produce the same results for the final --> step=10 line not taking into account the values in parenthesis which represents the execution time of the current iteration and the number of millions of cells processed by second. Those timings vary from machine to machine and are a good measurement of their respective performances.

| Hydro: OpenCL compute unit type = ACC |
|---|
| ++ |
| nx=2000 |
| ny=2000 |
| nxystep=128 |
| tend=1600.000 |
| |
| |
| |
| Hydro starts in double precision |
| Nh Distform - 1 |
| [0] Profile - FILL PROFILE |
| [0] VERSION : OpenCL 1.2 LINUX |
| [0] NAME : Intel(R) OpenCL |
| [0] VENDOR : Intel(R) Corporation |
| [0] EXTENSIONS : cl khr icd cl khr global int32 base atomics cl khr global int32 extended atomics |
| cl khr local int32 base atomics cl khr local int32 extended atomics cl khr byte addressable store cl khr spir |
| cl_khr_fp64 |
| (0) :: device maxcu 1 mxwkitdim 3 mxwkitsz 8192 8192 8192 mxwkgsz 8192 mxclockMhz 2100 mxmemallocsz 12057 |
| (Mo) globmemsz 48230 (Mo) type 2 [CPU] |
| extensions: cl_khr_icd cl_khr_global_int32_base_atomics cl_khr_global_int32_extended_atomics |
| cl_khr_local_int32_base_atomics_cl_khr_local_int32_extended_atomics_cl_khr_byte_addressable_store_cl_khr_spir |
| cl_intel_exec_by_local_thread_cl_khr_depth_images_cl_khr_3d_image_writes_cl_khr_fp64 |
| Device U supports double precision Hoatting point |
| prefered vector size: $C-1$ S-1 I-1 I-1 I-1 ($d-1$) |
| (1) .: UEVICE MARCU 250 MARKICUIM 5 MARKICUIM 5 MARKALS2 0152 0152 0152 MARKS2 0152 MARCUCKMM2 1052 MARMMAIDUS2 1524 |
| extensions: cl khr global int32 base atomics cl khr global int32 extended atomics |
| cl khr local int32 base atomics cl khr local int32 extended atomics cl khr byte addressable store cl khr spir |
| cl khr fp64 |
| Device 1 supports double precision floating point |
| prefered vector size: c=1 s=1 i=1 l=1 d=1 |
| Hydro: 000 has 0 GPU |
| Hydro: 000 has 1 ACC |
| Hydro: 000 uses ACC 0 |
| [0] : nbdevices = 2 |
| Building an ACC version |
| Build OpenCL (opts="-cl-mad-enable -DNVIDIA -DHASFP64 - |
| 1/ccc/gnome/ocre/collng/Gltnub/Hyaro/HyaroC/oclHyaroC_2D/Src ") OK. |
| Centerea Lest case : 1002 1002 |
| $\sum_{i=1}^{n} \frac{1}{2} \sum_{i=1}^{n} \frac{1}{2} \sum_{i$ |
| -> step=2 2.67261e=03, 1.33631e=03 (0.304s) |
| > step=3 5 70914-03, 3 03654-03 (0 361s) * |
| > step=4 8.74568-03, 3.03658-03 (0.301s) |
| > step=5 1.24942e-02, 3.74854e-03 (0.332s) * |
| > step=6 1.62428e-02, 3.74854e-03 (0.308s) (13.0 MC/s) |
| > step=7 2.06309e-02, 4.38811e-03 (0.333s) (12.0 MC/s) * |
| > step=8 2.50190e-02, 4.38811e-03 (0.301s) (13.3 MC/s) |
| > step=9 2.95700e-02, 4.55100e-03 (0.336s) (11.9 MC/s) * |
| > step=10 3.41210e-02, 4.55100e-03 (0.302s) (13.2 MC/s) |
| Hydro ends in 00:00:09.834s(9.834) without init: 3.258s. [DP] |
| GATCON CONPRI EOS SLOPE TRACE QLEFTR RIEMAN CMPFLX UPDCON COMPDT MAKBOU |
| |
| PEC 0.5///UU U.1661U4 U.1298/4 0.1/4436 0.365882 0.264103 0.5/1189 0.169403 0.587714 0.201799 |
| U.U.U.94030 U.UUUUUU § 17 017641 5 123022 / 005620 5 300016 11 204665 0 1/5553 17 616030 5 224790 10 126475 6 222060 |
| © 1/.01/041 3.123032 4.003023 3.300010 11.204003 0.143333 1/.010030 3.224/89 18.1264/5 0.223969 |
| |
| 1.051401 0.0000000 Average MC/s: 12.7 |

Table 4: output of the OpenCL version of Hydro using the KNC and one MPI task.

By comparing Table 3 and Table 4, we can see that the results are those expected.



| > step=9 | 2.95700e- | 02, 4.55100 |)e-03 (0.26 | 66s) (15.1 M | C/s) * | | | | |
|---------------|------------|-------------|-------------|--------------|----------|------------|----------|----------|----------|
| > step=10 | 3.41210e- | 02, 4.55100 |)e-03 (0.19 | 91s) (21.0 M | C/s) | | | | |
| Hydro ends in | 00:00:08.7 | 33s(8.733) | without ini | it: 2.406s. | [DP] | | | | |
| GATCON | CONPRI | EOS | SLOPE T | RACE QLE | FTR RIEM | IAN CMPFLX | UPDCON | COMPDT | MAKBOU |
| ALLRED | | | | | | | | | |
| MIN 0.316271 | 0.109199 | 0.085169 | 0.106515 | 0.200221 | 0.161903 | 0.318756 | 0.101318 | 0.306316 | 0.161775 |
| 0.395536 0.02 | 23942 | | | | | | | | |
| MAX 0.327046 | 0.111931 | 0.086332 | 0.117952 | 0.203285 | 0.163022 | 0.333114 | 0.108247 | 0.311196 | 0.219626 |
| 0.430098 0.10 | 03863 | | | | | | | | |
| AVG 0.321658 | 0.110565 | 0.085751 | 0.112234 | 0.201753 | 0.162462 | 0.325935 | 0.104782 | 0.308756 | 0.190701 |
| 0.412817 0.00 | 63903 | | | | | | | | |
| Average MC/s: | 18.2 | | | | | | | | |

Table 5: OpenCL run on a KNC using 2 MPI tasks on two separate nodes

Table 5 is the final validation of the system used in cluster mode. It shows that the results are unchanged and that therefore the MPI across nodes <u>and</u> KNCs is functional. Due to the hardware implementation of the communications between the KNC and its host, the poor speedup of the MPI version is no surprise. This situation should change dramatically with the forthcoming KNL.

Such tests demonstrate that:

- 1. The prototype system is functional on a per node basis as well as in a cluster mode
- 2. The performance of a single KNC is equivalent to the performance of a two sockets node (see the Average numbers in previous tables). Note that this is a small test case and the results may vary between programs and programming languages. Nonetheless, this gives very positive trends.

The KNC can also be used as standalone machine. This allows for better conditions to do code optimizations. For example, the behavior of directives can be investigated. In Figure 1 we wanted to see the impact of the directive #pragma simd on the output of Hydro in its OpenMP implementation. If a small number of iteration (10 to 1000) didn't show any significant differences, in the long run, rounding errors due to overaggressive optimizations can be noticed.



Figure 1: The first left third of the image is the reference run, the second image is the so called "optimized" run, the right third is the comparison between the two previous images and emphasizes the actual differences between the two runs, showing that the optimizations was not so great. The results should have been identical after 34000 time steps. Any difference in the pixels is denoted as a red pixel. Identical simulations will produce a pure white difference image.



3.4 Test of the CAPS developments

Because the CAPS partner had to withdraw from the project, no tests were made of their developments.

3.5 Test of the UVSQ developments

When writing D5.1.7.1, we were contemplating using the Hydro benchmark to evaluate MAQAO. In the meantime CEA encountered an optimization problem on another benchmark called Xpn. Xpn showed a major slow down when used on the MIC. Therefore, it was a good opportunity to tests the tools of the UVSQ partner while solving a real problem. The analysis described here has been done by the UVSQ partner and CEA is really grateful for the valuable insights it brought.

| BATAILLE - PROCESS #29251 |
|--|
| Thread #29251 - 29.66 second(s) |
| ****** |
| # Function Name Time % Time(s) CPI ratio Module # |
| ********** |
| # MPIDI CH3I Progress 94.07 27.90 0.77 libmpi.so.4.1 # |
| # spin lock 0.47 0.14 0.83 SYSTEM CALL # |
| $\#$ (\ldots) $\#$ |
| |
| BATAILLE - PROCESS #29250 |
| Thread $#29250 - 29.56$ second(s) |
| |
| # Function Name Time % Time(s) CPI ratio Module # |
| |
| # resc pn - 130@CALCUL.f 99.46 29.40 0.40 Xpn.AVX2 # |
| # spin lock 0.47 0.14 0.83 SYSTEM CALL # |
| # rb insert color 0.27 0.08 0.48 SYSTEM CALL # |
| ****** |
| BATAILLE - PROCESS #29249 |
| Thread $#29249 - 29.56$ second(s) |
| ******* |
| # Function Name Time % Time(s) CPI ratio Module # |
| |
| # resc pn - 130@CALCUL.f 99.59 29.44 0.40 Xpn.AVX2 # |
| # spin lock 0.47 0.14 0.83 SYSTEM CALL # |
| **** |
| BATAILLE - PROCESS #29248 |
| Thread $#29248 - 29.56$ second(s) |
| |
| # Function Name Time % Time(s) CPI ratio Module # |
| ************ |
| # resc pn - 130@CALCUL.f 99.86 29.52 0.40 Xpn.AVX2 # |
| # spin lock 0.47 0.14 0.83 SYSTEM CALL # |
| ****** |
| |

Table 6: profile of the Xpn benchmark using 4 MPI tasks

The first step was to determine where all the time was spent (Table 6). Fortunately, it was in a single routine (resc_pn) which eased the next steps.

The structure of resc_pn looks like (Table 7):



| do id=1,x |
|-----------------------|
| <some code=""></some> |
| innermost loop #1 |
| <some code=""></some> |
| innermost loop #2 |
| <some code=""></some> |
| innermost loop #3 |
| <some code=""></some> |
| innermost loop #4 |
| <some code=""></some> |
| end do |
| |

Table 7 : structure of the resc_pn subroutine

The result of a profiling session (Table 8) on Xpn shows that most of the time is spent outside the inner loops.

| ## | ######### | **** | **** |
|----|-----------|---|----------------------|
| # | Loop ID | Source Infos Level Time % | Time (s) CPI ratio |
| ## | ######## | ***** | ***** |
| # | 454 | resc pn - 982,1038@CALCUL.f InBetween 3.18 | 0.94 0.51 |
| # | 341 | resc pn - 1447,1504@CALCUL.f InBetween 3.18 | 0.94 0.58 |
| # | 416 | resc_pn - 1141,1198@CALCUL.f InBetween 3.04 | 0.90 0.58 |
| # | 227 | resc pn - 1895,1947@CALCUL.f InBetween 2.98 | 0.88 0.50 |
| # | 245 | resc pn - 1830,1883@CALCUL.f InBetween 2.91 | 0.86 0.46 |
| # | 530 | resc_pn - 668,724@CALCUL.f InBetween 2.84 | 0.84 0.46 |
| # | 568 | resc pn - 511,568@CALCUL.f InBetween 2.84 | 0.84 0.53 |
| # | 626 | resc_pn - 268,324@CALCUL.f InBetween 2.64 | 0.78 0.49 |
| # | 550 | resc_pn - 581,637@CALCUL.f InBetween 2.64 | 0.78 0.45 |
| # | 587 | resc pn - 432,488@CALCUL.f InBetween 2.57 | 0.76 0.50 |
| # | 493 | resc_pn - 817,874@CALCUL.f InBetween 2.57 | 0.76 0.44 |
| # | 397 | resc pn - 1221,1277@CALCUL.f InBetween 2.37 | 0.70 0.45 |
| # | 644 | resc pn - 197,255@CALCUL.f InBetween 2.37 | 0.70 0.42 |
| # | 379 | resc_pn - 1291,1348@CALCUL.f InBetween 2.30 | 0.68 0.42 |
| # | 607 | resc pn - 348,405@CALCUL.f InBetween 2.30 | 0.68 0.45 |
| # | 283 | resc pn - 1685,1738@CALCUL.f InBetween 2.30 | 0.68 0.37 |
| # | 208 | resc_pn - 208,2021@CALCUL.f InBetween 2.30 | 0.68 0.39 |
| # | 512 | resc pn - 736,793@CALCUL.f InBetween 2.23 | 0.66 0.35 |
| # | 302 | resc_pn - 1612,1665@CALCUL.f InBetween 2.23 | 0.66 0.39 |
| # | 436 | resc_pn - 1050,1107@CALCUL.f InBetween 2.17 | 0.64 0.34 |
| # | 359 | resc pn - 1377,1433@CALCUL.f InBetween 2.17 | 0.64 0.38 |
| # | 265 | resc_pn - 1750,1802@CALCUL.f InBetween 2.03 | 0.60 0.31 |
| # | 473 | resc pn - 902,958@CALCUL.f InBetween 1.83 | 0.54 0.31 |
| # | 322 | resc pn - 1526,1582@CALCUL.f InBetween 1.69 | 0.50 0.33 |
| # | 459 | resc_pn - 1021,1023@CALCUL.f Innermost 0.68 | 0.20 0.76 |
| # | 577 | resc_pn - 536,538@CALCUL.f Innermost 0.68 | 0.20 0.79 |
| # | 421 | resc_pn - 1181,1183@CALCUL.f Innermost 0.61 | 0.18 1.09 |
| | | | |

Table 8 : detailed profiling of resc_pn showing that most of the time is not spent in the innermost loops.

Since MAQAO was, by design, focusing on inner loops, it had to be extended to be able to cope with this situation, producing a richer tool. From this extended version, many lessons were learned on Xpn.

First, most of the time is spent on not vectorized section (peel or tail as explained in Table 9). This means that the loops do not have enough indices to work on.



```
// peel loop, to align on vector boundaries
do i=2,2 body(i) end do
// main vector loop, unrolled x4
do i=3,18 (step 16)
    body(vector[i+ 0 to i+ 3])
    body(vector[i+ 4 to i+ 7])
    body(vector[i+ 8 to i+11])
    body(vector[i+12 to i+15])
end do
// main vector loop, not unrolled
do i=19,22 (step 4)
    body(vector[i to i+3])
end do
// tail loop, to handle leftover elements
do i=22,23 body(i) end do
```

Table 9 : classical decomposition of a loop by a compiler.

Second, the quality of the generated code is not optimal as the CQA tool proves it in Table 10.

| ASM code | |
|--|--|
| ASM code MOV MOV MOV MOV VMOVUPD VMOVUPD VMOVUPD VMOVUPD VMOVUPD VMOVUPD VMULPD VFMADD132PD VFMADD132PD VFMADD132PD VFMADD132PD VFMADD132PD VFMADD132PD VFMADD132PD VFMADD132PD VFMADD132PD VFMADD132PD VFMADD132PD VFMADD132PD VFMADD132PD VFMADD132PD VFMADD132PD VFMADD132PD VFMADD132PD VFMADD132PD | 0x1460(%RSP),%RBX 0x1450(%RSP),%RAX 0x1470(%RSP),%RCX 0x1468(%RSP),%RDX 0x60(%RAX,%RBX,8),%YMM5 0x40(%RAX,%RBX,8),%YMM6 0x20(%RAX,%RBX,8),%YMM8 0xd40(%RSP),%YMM3 (%RAX,%RBX,8),%YMM9 0x60(%RCX,%RBX,8),%YMM5,%YMM7 0x40(%RCX,%RBX,8),%YMM5,%YMM3 0x40(%RCX,%RBX,8),%YMM6,%YMM3 0x40(%RCX,%RBX,8),%YMM6,%YMM6 %YMM8,%YMM4 0x20(%RCX,%RBX,8),%YMM6,%YMM8 0x20(%RCX,%RBX,8),%YMM6,%YMM8 0x20(%RCX,%RBX,8),%YMM6,%YMM8 0x20(%RCX,%RBX,8),%YMM6,%YMM4 %YMM9,%YMM1 (%RCX,%RBX,8),%YMM0,%YMM1 (%RCX,%RBX,8),%YMM0,%YMM1 (%RCX,%RBX,8),%YMM1,%YMM2 0xd60(%RSP),%YMM1,%YMM2 0xd60(%RSP),%YMM10 %YMM1,0xd40(%RSP) %YMM10,0xd60(%RSP) |
| ADD MOV CMP JB | <pre>\$UX10,%RBX %RBX,0x1460(%RSP) 0x1478(%RSP),%RBX 417ad5 <resc +0x3555="" pn=""></resc></pre> |
| | |

Table 10 : generated assembly code using the default (-O3) optimizing options.

The not vectorized version (option -no-vec) was much cleaner (Table 11) and surprisingly 2X faster.



| ASM code | | |
|---------------|--|--|
| VMOVSD | (%RCX,%RBP,8),%XMM15 | |
| VMULSD | (%RDX,%RBP,8),%XMM15,%XMM2 | |
| VADDSD | %XMM3,%XMM2,%XMM3 | |
| VMULSD | (%RAX,%RBP,8),%XMM15,%XMM2 | |
| INC | %RBP | |
| VADDSD | %XMM4,%XMM2,%XMM4 | |
| CMP | %R13 , %RBP | |
| JB | 41ab28 <resc_pn_+0x7278></resc_pn_+0x7278> | |

Table 11 : assembly code of the loop forcing the not vectorized mode.

The best vectorized code was produced using an option (-unroll0) to refrain the compiler to over optimize the final binary. It produces a clean assembly as shown in Table 12 yet it is only 1.9X faster due to the limited span of the inner loops:

| ASM cod | ASM code | | |
|---|---|--|--|
| VMOVUPD VMULPD VMULPD VADDPD VADDPD ADD CMP | - (%RDX,%RAX,8),%YMM4 (%RDI,%RAX,8),%YMM4,%YMM3 (%R10,%RAX,8),%YMM4,%YMM12 %YMM2,%YMM3,%YMM2 %YMM1,%YMM12,%YMM1 \$0x4,%RAX %RSI,%RAX | | |
| JB | 414e89 <resc +0x13e9="" pn=""></resc> | | |

Table 12 : assembly code of the loop with moderate vectorization

This statement is confirmed by the static analysis (Table 13) of the resource usage of the binary where P0... P7 are the internal paths (named ports) to functional units of the microprocessor. P0 and P1 are devoted to floating point operations. P2 and P3 are devoted to loads and stores. A compute routine must spend most of its cycles in P0 or P1 otherwise it is memory bound.



| Back-end | | | | | | | | |
|--|--|--|--|--|--|--|----------------------------------|--------------------|
| Uops Cycles Cycles Longest Cycles | P0 2.00 2.00 execu recu summa | P1 2.00 2.00 ting d rrence ry | P2 1.50 1.50 liv or e chain | P3 1.50 1.50 sqrt i: laten | P4 0.00 0.00 nstruc cy (Re | P5 1.00 1.00 tions: cMII): | P6 1.00 1.00 NA 3.00 | P7 0.00 0.00 |
| Front-end Dispatch Data deps. Max Vector efficienc | | | 2.25 2.00 3.00 3.00 ratios | | | | | |
| all load store mul add sub | : 10 : 10 : NA : 10 : 10 | 0% 0% (no s 0% 0% | store v | ectori | zable/ | vector | ized i | nstructions) |
| other | : NA | (no c | ther v | ectori | zable/ | vector | ized i | nstructions) |

Table 13 : static analysis of the binary showing what functional units are used.

All those analysis can be summarized as

- 1. Xpn performance was limited by the complexity of the loop nest and especially the "in-between sections".
- 2. The compiler may take wrong decisions to optimize a code and using MAQAO can help to verify that the proper strategy was taken. If not then explore other non-classical approaches such as the -unrollo option.
- 3. Consider reorganizing the code in depth to promote longer vectors and optimize data locality.

Using the previous remarks and a further restructuring, the Xpn benchmark was accelerated by a factor of 4, a major step forward, which demonstrates the importance of tools such as MAQAO to understand where the bottlenecks are and to verify that the applied fix did solve the problem.

3.6 Usage of the prototype

The prototype has been extensively used either internally by CEA/DAM or by the PERFCLOUD partners. As a measure of such activity, the total number of minutes used on the prototype for year 2014 is 1875909 minutes, which is an equivalent of 144 days of continuous work per node (out of 192 working days for CEA staff member). This level of usage is a clear demonstration that this prototype is a very successful machine as the next section will illustrate also.

Figure 2 : the Cirrus prototype

4 A more elaborate benchmark

4.1 Relations with the initial H4H project

DA participated as a full funded partner in the initial period – from October 2010 to October 2013 – of the H4H project. As reported in deliverable D4.1.5 (see [D4.1.5]), the bulk of the work accomplished during this period was centered on the adaptation of DA's simulation tools for aircraft design on many-core architectures: at that time, as MIC was in its infancy, NVIDIA's GPUs were the only readily available many-core hardware.

Amid several successful works on GPU-adaptation, there stood out one case in which multiple GPUoptimization efforts gave no meaningful results: AeTHER, our CFD software based on FEM formulation and unstructured meshes discretization, in which the FEM matrix & residual assembly is the main numerical kernel. Several reasons to explain this situation were provided, which are all somehow related to AeTHER's data-bound behavior; the two main ones are:

- 1) On hybrid CPU-GPU hardware with a PCIe link to connect GPU with CPU the numerical intensity at the elemental level is not high enough to offset the cost of data transfer, even when element blocking is activated.
- 2) Even on standard CPUs multi-core nodes, our *legacy* shared-memory parallel implementation, which is based on element multi-coloring techniques, has a poor scalability for numbers of cores over 3 or 4.

Progress on the first point would require substantial hardware improvement, which seemed out of scope in the H4H project. To improve on the second point, we need to develop <u>a scalable shared</u> <u>memory parallelization technique for the FEM matrix and residual assembly kernel on unstructured</u> <u>meshes</u>: that's exactly what we did in the second half of the H4H project – from June 2012 to October 2013 - in collaboration with the UVSQ partner, during a POC (Proof Of Concept) action to demonstrate the effectiveness of the <u>Divide and Conquer</u> (D&C) approach (see [D4.1.5]).

In the last year of the H4H/Perfcloud project, as MIC architecture became more available, DA embarked on 2 actions, which were carried out in parallel:

1) Internally in DA, gradual integration of D&C in AeTHER, on standard CPUs multi-core architecture.

2) In the H4H/Perfcloud project, test of the D&C approach on the MIC architecture, which is available on the Cirrus prototype.

In what follows, we will first recall AeTHER'S MPI implementation and some of the D&C integration work, as well as how the FEM mini-application was derived. Finally, we will report on the benchmark results on using MIC architecture for the FEM mini-application, running with industrial-grade unstructured meshes.

4.2 Technical description: a CFD software and its derived FEM mini-application

4.2.1 Introducing D&C in AeTHER

The reader is referred to deliverable D4.1.1 (see [D4.1.1]) for a detailed description of AeTHER, DA's CFD software based on FEM formulation and unstructured meshes discretization. Moreover, as mentioned above, an extensive description of the D&C approach can be found in deliverable D4.1.5 (see [D4.1.5]). At this point, we would just recall the basic facts leading to a hybrid MPI + D&C parallel approach, which we would like to implement in AeTHER.

As depicted on the unstructured mesh shown on the left part of Figure 3, AeTHER solves a system of non-linear equations with the following FEM features:

- The unknowns and the equations also called *residuals* in FEM terminology are located at the mesh nodes.
- In each element, the evaluation of the non-linear residuals is done with unknown values read in from the element nodes and the results are stored back at the nodes.

A quasi-Newton iterative algorithm, based on linearization procedures, is used for the solution of the non-linear residual equations: it requires the construction of a sparse matrix which is built-up from local elemental matrices computed from local linearization of the residual equations. In short, at each non-linear iteration, AeTHER's work load consists in:

- Kernel 1, FEM residual and matrix assembly: it's usually the most compute-intensive part, with a number of Flops ≈ O (number of elements).
- Kernel 2, Solution of the linearized equations: an iterative GMRES algorithm is used here and, in *quasi-Newton fashion*, only a limited number of linear iterations is necessary. Thus this kernel is fairly compute-intensive with a number of Flops ≈ O (number of nodes).

For a MPI parallelization of AeTHER, it's reasonable to focus on kernel 1 and use no-element overlap partitioning of meshes to distribute the load on separate MPI processes working in parallel: this procedure is depicted on the right part of Figure 3, where one can see the 1-layer-node overlap at the interfaces, with the corresponding MPI message exchanges to recover the global residual values at these nodes.

Figure 3: MPI implementation via mesh partitioning. Left: sequential. Right: MPI.

To sum up, this MPI parallel implementation has the following features:

- 1) For kernel 1: the work load is very well balanced, as much as the mesh partitions are balanced in terms of number of elements.
- 2) For kernel 2: the work load is fairly balanced (when we compare 2 unstructured meshes, an equal number of elements does not guarantee an equal number of nodes...). Moreover there is a small overhead in terms of number of nodes, corresponding to the 1-layer-node overlap.

As one may suspect, this MPI-only (or *flat* MPI) model does not scale well when the number of physical cores (and corresponding MPI processes...) grows over the 500s. In addition to the growing complexity of the MPI messages and the sheer difficulty in process synchronization, the overhead in interfaces nodes for kernel 2 may become sizable: e.g. for an industrial-grade unstructured mesh of 7 Million-nodes, partitioned in 1024 mesh blocks, this overhead reaches 24% of the total number of nodes.

In DA, the above observations were made several years ago and since then, numerous efforts were done to keep a reasonable number of mesh blocks (and corresponding MPI processes) by introducing shared memory parallelism inside each MPI process, in short a hybrid MPI + threads model.

A legacy procedure (implemented in AeTHER) is to re-factor element *multi-coloring* techniques, which were used for vectorization, to parallelize the loop over elements in kernel 1: this is shown on the left part of Figure 4, where one can see the scattered elements, all having the same color, which are then processed in parallel threads. One observation can be made, which is the usual warning of cache-aware optimization experts about coloring techniques: one did introduce a lot of concurrency but at the expense of almost random memory access, which is very bad for performance on multi-cache multi-core hardware.

Figure 4: MPI block-wise shared memory implementations. Left: element coloring. Right: D&C recursive partitioning.

At the bottom line, the D&C approach continues the mesh partitioning, in a recursive manner, inside each MPI mesh block. This is depicted in the right half of Figure 4, where the left mesh block of the MPI partitioned mesh in Figure 3 is further divided into 2 mesh sub-blocks (red and green) separated by a mesh sub-block (blue). This procedure can be further applied to each mesh sub-block, and so on...This D&C shared memory parallel implementation has the following nice features:

- Data locality: the data processed by each thread corresponds to a mesh block (i.e. a contiguous group of elements), which means that, provided element and node renumbering are done, we have data locality at will...small enough to fit into L3 caches.
- No further interface nodes overhead: in a shared memory framework and with the (blue) mesh separator sub-block, there is no additional node overhead due to mesh partitioning.
- Local synchronizations: at one given level of D&C partitioning, local synchronization is required only to ensure that the red and green computations are done in parallel before the blue computations. The full organization of the different parallel tasks is done via a tree-structured task graph, as depicted in the right half of Figure 5.

In the flat-MPI parallel model, as there is only one level of synchronization (MPI process), the synchronization actions appear to be global, i.e. small mesh blocks have to wait for larger mesh blocks even if they are not connected together: there is not much room for asynchronicity in this very "democratic" organization, where every MPI process has the same priority.

In the D&C parallel model, there are several successive levels of local synchronizations, which allows for asynchronous behavior. Unlike the flat-MPI "democratic" model, the effective model here is "work stealing": a limited number of parallel workers (corresponding to the physical cores) pick out work in a much larger pool of parallel tasks in an order defined by the task graph.

Figure 5: Cilk recursive implementation of D&C work stealing parallel model.

On Intel-based platforms, this work-stealing model can be effectively implemented via the Cilk API. This is shown on the left part of Figure 5, with a model source code, which includes Cilk commands and recursive function calls. On other platforms, D&C can be in theory implemented in the OpenMP API: this is on-going development work.

As mentioned above, in the last year of the H4H/Perfcloud project, D&C was introduced in the MPIparallel implementation of AeTHER: more precisely, D&C was activated only for kernel 1. On Intelbased Bullx nodes, successful runs of D&C-equipped AeTHER on industrial-grade unstructured meshes of sizes 1 to 7 Million nodes have shown the following:

- 1) For a fixed number of cores, D&C permits faster runs with fewer MPI blocks (64 MPIs vs. 16 MPIs x 4 threads).
- 2) For a fixed number of MPI blocks, D&C renumbering without thread parallelism permits faster runs, even when comparisons are made against the MPI + vectorization version. One possible explanation is the much reduced number of cache misses, as measured by the MAQAO tool in Figure 6.

Figure 6: Measuring caches misses with MAQAO for CFD runs on a 1 M-nodes mesh

4.2.2 The FEM mini-application

As detailed in deliverable D4.1.5 (see [D4.1.5]), the POC action to validate the D&C approach was the result of a collaborative action between DA and UVSQ, working on DefMesh, a FEM software

representative of AeTHER and which uses the same unstructured meshes. In addition to software confidentiality issues, the other reason for this procedure was to work on a much simpler code, which still possesses all the implementational complexities related to FEM and unstructured meshes.

In the last year of the H4H/Project, DA has pursued work along these lines by developing, with the very active collaboration of UVSQ (in the framework of the EXA2CT project, see [EXA2CT]), a FEM mini-application which is software extracted from DefMesh, with the following features:

- 1) It uses the same unstructured meshes as DefMesh (and AeTHER).
- 2) It implements the FEM matrix and residual assembly kernel, for the linear elasticity operator.

Compared with the full Navier-Stokes operator in AeTHER, we're quite far away in terms of numerical intensity and memory footprint. Nevertheless, all the FEM data localization and reshuffling are present, and, in light of our recent work introducing D&C into AeTHER after the POC action, we can assure that the benefits we get with a simplified operator will be greater with a more complex operator, requiring more *local computations* but on the *same unstructured mesh*.

4.3 Benchmark results on the FEM mini-application

The benchmark operation was done with the following features:

- 1) Software: FEM mini-application defined above.
- 2) Platform: MIC accelerator component of Cirrus prototype.
- 3) Mesh data: industrial-grade 7 Million nodes mesh, around a generic Falcon aircraft.

Figure 7: Mesh views of a Falcon type civil aircraft. Left: general 3D view. Right: zoom on surface meshes near the cockpit.

Mesh views are shown in Figure 7, where the zoom on surface meshes near the aircraft cockpit shows that practically all the nodes are concentrated near the aircraft surface in order to properly simulate the boundary layer. More precisely, 80% of the nodes are located at a distance of less than 0.5 meters from the aircraft surface, whereas the far-field surface (the outermost surface of the discretized volume around the aircraft) is about one kilometer (1000 meters) from the aircraft surface. These figures show the highly unstructured nature of the mesh, with several orders of magnitude in geometric scales.

Despite these highly irregular features, the application of D&C recursive partitioning results in a clustering effect on the numbering of nodes and elements, leading to the desired data locality. One way to measure this data locality is to compute the maximum distance, in terms of node numbers, between the nodes of an element (|i - j| in left part of Figure 8): the smaller this distance is, the better one can squeeze all the element data inside L3 cache, thus removing potential cache misses.

In the right half of Figure 8, cumulated counts of this distance on the whole set of mesh elements is shown. As highlighted by the green line, representing a distance value of 10,000:

- For the initial element and node numbering of the mesh, **almost 50%** of the elements have a maximum distance of **more than 10,000**.
- For the mesh with D&C element and node renumbering, **only 5%** of the elements have a maximum distance of **more than 10,000**.

Figure 8: Data localization induced by D&C clustering

For the runs on the MIC/KNCs of Cirrus, we are comparing 2 versions of the FEM mini-application: flat-MPI and MPI + DC (Cilk). To accommodate a mesh of this size (7 Million nodes), we used **4 KNCs throughout the runs**: the resulting MPI runs will mix MPI intra-communications (between the 60 cores of a KNC) with MPI inter-communications (between cores of different KNCs, i.e. via the PCIe link...).

For the flat-MPI version, the following mesh partitions were prepared: 4, 128, 256 and 512 blocks.

For the MPI + DC (Cilk) version, because of memory limitations, we start with 4 MPIs (one on each KNC) and increase the number of threads: 4×1 , 4×30 , 4×60 , 4×120 , 4×180 , 4×240 .

As the number of available physical cores is 4×60 (= 240), any run with a higher number of cores activates hyper threading.

In Figure 9, scalability results of the benchmark runs are shown:

- Parallel efficiency on the left part: MPI+D&C clearly outperforms flat-MPI, with a parallel efficiency of more than 90% for the full number of physical cores, 4 x 60 = 240.
- Speed-up on the right part, with the 4 MPIs x 1 D&C run as reference: once again, MPI+D&C outperforms flat-MPI, by a factor of more than 2 for the full number of physical cores, 4 x 60 = 240.

Figure 9: Scalability results with 4 KNCs. Left: parallel efficiency. Right: speed-up.

To be fair, the flat-MPI runs may be hindered by the growing MPI inter-communications between KNCs, as compared with the MPI+D&C runs which are limited to inter-communications between 4 MPI processes distributed on 4 KNCs (1 MPI on each KNC).

In previous tests with a smaller size mesh (1 Million nodes), described in [PPoPP15], we were able to fit into one KNC of Cirrus, thus using only MPI intra-communications. The comparisons were still in favor of the MPI+D&C version, but with a somewhat lesser difference.

5 Abbreviations and acronyms

MIC[®]: Many Integrated Core. The public brand name is Intel Xeon Phi and describes processors having a lot of cores within its chip.

KNC: (Knights corner) Intel's code name of the Xeon Phi.

#pragma simd: a directive provided by a programmer to tell the compiler to use special hardware functional units operating on a number of operands at the same time (SIMD stands for **S**ingle Instruction Multiple Data).

CFD: Computational Fluid Dynamics, for the numerical simulation of fluid flows.

FEM: Finite Element Method.

PCIe: PCI-eXpress, standard (Intel) solution to link CPU hardware with its many-core component (GPUs, MIC) on a node.

D&C: Divide and Conquer approach for shared memory parallelization. It has been successfully applied to the FEM matrix & residual assembly kernel, on fully unstructured 3D meshes.

GMRES: General Minimal RESidual. An effective (and popular) iterative algorithm for the solution of systems of sparse linear equations, with a general definite sparse matrix.

Cilk: An Intel API to implement shared memory thread parallelism. It's particularly efficient for task-based parallelism.

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